## Collaborative Project - Limits and an Introduction to Calculus

The graph shows the profit that a new computer service business generates during its first 10 years of business.


1. Let $x$ represent the year.
a. Use the regression feature of a graphing utility to find a cubic function $p$ for the profit.
b. Estimate the slope of the graph when $x=4$ and when $x=9$ and interpret the results.
c. Do you think the owners of the company feel more confident about the business in year 5 or year 8? Explain.
d. Find the derivative of the function. Use your answer to check the accuracy of your estimations in part (a).
e. Find the limit of the function as $x$ approaches infinity and interpret the result. Is the result meaningful in this situation? Explain.
2. The owners of the company want to increase their profit by a consistent amount every year or at least maintain a steady annual profit every year.
a. List some types of functions that approach a constant value or approach constant rate of change as $x$ approaches infinity.
b. Suppose the company wants their profit to approach $\$ 100,000$ per year. Give an example of a rational function that has a limit of 100 as $x$ approaches infinity. The function does not need to actually model the profit.
c. Suppose the company wants their profit to approach an increase of $10 \%$ every year. Give an example of a rational function that approaches the line $y=1.1 x$ as $x$ approaches infinity. The function does not need to actually model the profit. Does the function have a limit as $x$ approaches infinity?
3. Approximate the total profit by finding the area of each region.
a. The region bounded by the graph of the function $p$ in Exercise 1, the $x$-axis, and the lines $x=0$ and $x=10$.
b. The region bounded by the graph of the function $p$, the $x$-axis, and the lines $x=0.5$ and $x=10.5$.

Which approximation is better? Explain.

