## Collaborative Project - Polynomial Functions

1. Creative Candles is testing the burn time of a hemisphere-shaped soy candle with a base diameter of 6 inches. Burning continuously, the candle does not tunnel, so its melted top surface is flat. The table shows the burning candle's height (in inches) every 2 hours.

| Time, $t$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Height, $h$ | 3 | 2.24 | 1.90 | 1.39 | 1.18 | 0.97 | 0.78 | 0.59 | 0.41 |

a. Use a graphing utility to create a scatter plot of the data. Then use the regression feature of the graphing utility to find a quadratic model and a cubic model for the data. Graph both models with the data. Discuss how well each model fits the data.
b. Find the minimum value of the quadratic model. Does this function have any real zeros? Any complex zeros? Explaining your reasoning.
c. Will the quadratic model accurately predict the candle's height after 20 hours? Explain.
d. Describe the left-hand and right-hand behavior of the graph of the cubic function.
e. Explain why the cubic function must have at least one real zero.
2. Creative Candles wants to design an open-top box that can hold several different candles. The box will be formed by cutting squares of length $x$ from the corners of a rectangular piece of cardboard 12 inches wide and 15 inches long, and turning up the sides.
a. Write a function $V$ that represents the volume of the box and find the domain.
b. Use the table feature of a graphing utility to approximate the zeros of the function $f(x)=V(x)-150$ to find the side lengths $x$ that result in a volume of 150 cubic inches.
3. A model for the relationship between the annual advertising expenses $x$ and the profits $P$ for Creative Candles is $P(x)=-0.2 x^{3}+1.4 x^{2}+11.4 x^{2}-20$, where $x$ and $P$ are both in tens of thousands of dollars. The company wants to expand its operations this year, while earning a profit of at least $\$ 340,000$.
a. Use a graphing utility to graph the function $P$. Use the graph to approximate the two annual advertising amounts that give the minimum amount of desired profit.
b. Explain how you can use synthetic division to verify the two advertising amounts in part (a). Then use synthetic division to verify the greater advertising amount.
4. Creative Candles has interns that sell candles door to door. On average, daily door-to-door sales vary jointly with the number of houses visited and the median household income of the neighborhood. When 138 houses were visited in a neighborhood with a median household income of $\$ 56,000$, the amount of candles sales in one day was $\$ 2475$.
a. Write an equation relating the number of houses visited $n$, median household income $h$ (in thousands of dollars), and daily sales $s$ (in dollars).
b. Estimate the daily door-to-door sales in a neighborhood when 95 houses are visited in a neighborhood with a median household income of $\$ 62,000$.

