Appendix E: Solving Linear Equations and Inequalities

What you should learn

- Solve linear equations in one variable.
- Solve linear inequalities in one variable.

Why you should learn it

The method of solving linear equations is used to determine the intercepts of the graph of a linear function. The method of solving linear inequalities is used to determine the domains of different functions.

Linear Equations

A *linear equation* in one variable x is an equation that can be written in the standard form $ax + b = 0$, where a and b are real numbers with $a \neq 0$.

A linear equation in one variable, written in standard form, has exactly one solution. To see this, consider the following steps. (Remember that $a \neq 0$.)

$ax + b = 0$ Original equation

$ax = -b$ Subtract b from each side.

$x = -\dfrac{b}{a}$ Divide each side by a.

To solve a linear equation in x, isolate x on one side of the equation by creating a sequence of *equivalent* (and usually simpler) equations, each having the same solution(s) as the original equation. The operations that yield equivalent equations come from the Substitution Principle and the Properties of Equality.

Generating Equivalent Equations

An equation can be transformed into an *equivalent equation* by one or more of the following steps.

	Original Equation	*Equivalent Equation*
1. Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the equation.	$2x - x = 4$	$x = 4$
2. Add (or subtract) the same quantity to (from) *each* side of the equation.	$x + 1 = 6$	$x = 5$
3. Multiply (or divide) *each* side of the equation by the same *nonzero* quantity.	$2x = 6$	$x = 3$
4. Interchange the two sides of the equation.	$2 = x$	$x = 2$

After solving an equation, check each solution in the original equation. For example, you can check the solution of the equation in Step 2 above as follows.

$x + 1 = 6$ Write original equation.

$5 + 1 \stackrel{?}{=} 6$ Substitute 5 for x.

$6 = 6$ Solution checks. ✓

EXAMPLE 1 Solving Linear Equations

a.

$3x - 6 = 0$	Original equation
$3x - 6 + 6 = 0 + 6$	Add 6 to each side.
$3x = 6$	Simplify.
$\dfrac{3x}{3} = \dfrac{6}{3}$	Divide each side by 3.
$x = 2$	Simplify.

b.

$4(2x + 3) = 6$	Original equation
$8x + 12 = 6$	Distributive Property
$8x + 12 - 12 = 6 - 12$	Subtract 12 from each side.
$8x = -6$	Simplify.
$x = -\dfrac{3}{4}$	Divide each side by 8 and simplify.

Linear Inequalities

Solving a linear inequality in one variable is much like solving a linear equation in one variable. To solve the inequality, you isolate the variable on one side using transformations that produce *equivalent inequalities*, which have the same solution(s) as the original inequality.

Generating Equivalent Inequalities

An inequality can be transformed into an *equivalent inequality* by one or more of the following steps.

	Original Inequality	*Equivalent Inequality*
1. Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the inequality.	$4x + x \geq 2$	$5x \geq 2$
2. Add (or subtract) the same number to (from) *each* side of the inequality.	$x - 3 < 5$	$x < 8$
3. Multiply (or divide) each side of the inequality by the same *positive* number.	$\frac{1}{2}x > 3$	$x > 6$
4. Multiply (or divide) each side of the inequality by the same *negative* number and *reverse* the inequality symbol.	$-2x \leq 6$	$x \geq -3$

EXAMPLE 2 Solving Linear Inequalities

a. $x + 5 \geq 3$ — Original inequality

$x + 5 - 5 \geq 3 - 5$ — Subtract 5 from each side.

$x \geq -2$ — Simplify.

The solution is all real numbers greater than or equal to -2, which is denoted by $[-2, \infty)$. Check several numbers that are greater than or equal to -2 in the original inequality.

b. $-4.2m > 6.3$ — Original inequality

$\frac{-4.2m}{-4.2} < \frac{6.3}{-4.2}$ — Divide each side by -4.2 and reverse inequality symbol.

$m < -1.5$ — Simplify.

The solution is all real numbers less than -1.5, which is denoted by $(-\infty, -1.5)$. Check several numbers that are less than -1.5 in the original inequality.

Remark

Remember that when you multiply or divide by a negative number, you must *reverse the inequality symbol*, as shown in Example 2(b).

E Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises. For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blank.

1. A _______ equation in one variable x is an equation that can be written in the standard form $ax + b = 0$.
2. To solve a linear inequality, isolate the variable on one side using transformations that produce _______ .

Procedures and Problem Solving

Solving Linear Equations **In Exercises 3–24, solve the equation and check your solution.**

3. $x + 11 = 15$
4. $x + 3 = 9$
5. $x - 2 = 5$
6. $x - 5 = 1$
7. $3x = 12$
8. $2x = 6$
9. $\frac{x}{5} = 4$
10. $\frac{x}{4} = 5$
11. $8x + 7 = 39$
12. $12x - 5 = 43$
13. $24 - 7x = 3$
14. $13 + 6x = 61$
15. $8x - 5 = 3x + 20$
16. $7x + 3 = 3x - 17$
17. $-2(x + 5) = 10$
18. $4(3 - x) = 9$
19. $2x + 3 = 2x - 2$
20. $8(x - 2) = 4(2x - 4)$
21. $\frac{3}{2}(x + 5) - \frac{1}{4}(x + 24) = 0$
22. $\frac{3}{2}x + \frac{1}{4}(x - 2) = 10$
23. $0.25x + 0.75(10 - x) = 3$
24. $0.60x + 0.40(100 - x) = 50$

Solving Linear Inequalities **In Exercises 25–46, solve the inequality and check your solution.**

25. $x + 6 < 8$
26. $3 + x > -10$
27. $-x - 8 > -17$
28. $-3 + x < 19$
29. $6 + x \leq -8$
30. $x - 10 \geq -6$
31. $\frac{4}{5}x > 8$
32. $\frac{2}{3}x < -4$
33. $-\frac{3}{4}x > -3$
34. $-\frac{1}{6}x < -2$
35. $4x < 12$
36. $10x > -40$
37. $-11x \leq -22$
38. $-7x \geq 21$
39. $x - 3(x + 1) \geq 7$
40. $2(4x - 5) - 3x \leq -15$
41. $7x - 12 < 4x + 6$
42. $11 - 6x \leq 2x + 7$
43. $\frac{3}{4}x - 6 \leq x - 7$
44. $3 + \frac{2}{7}x > x - 2$
45. $3.6x + 11 \geq -3.4$
46. $15.6 - 1.3x < -5.2$