## **Collaborative Project – Limits and an Introduction to Calculus**

The graph shows the profit that a new computer service business generates during its first 10 years of business.



- **1.** Let *x* represent the year.
  - **a.** Use the regression feature of a graphing utility to find a cubic function p for the profit.
  - **b.** Estimate the slope of the graph when x = 4 and when x = 9 and interpret the results.
  - **c.** Do you think the owners of the company feel more confident about the business in year 5 or year 8? Explain.
  - **d.** Find the derivative of the function. Use your answer to check the accuracy of your estimations in part (a).
  - **e.** Find the limit of the function as *x* approaches infinity and interpret the result. Is the result meaningful in this situation? Explain.
- **2.** The owners of the company want to increase their profit by a consistent amount every year or at least maintain a steady annual profit every year.
  - **a.** List some types of functions that approach a constant value or approach constant rate of change as *x* approaches infinity.
  - **b.** Suppose the company wants their profit to approach 100,000 per year. Give an example of a rational function that has a limit of 100 as *x* approaches infinity. The function does not need to actually model the profit.
  - **c.** Suppose the company wants their profit to approach an increase of 10% every year. Give an example of a rational function that approaches the line y = 1.1x as x approaches infinity. The function does not need to actually model the profit. Does the function have a limit as x approaches infinity?
- 3. Approximate the total profit by finding the area of each region.
  - **a.** The region bounded by the graph of the function p in Exercise 1, the *x*-axis, and the lines x = 0 and x = 10.
  - **b.** The region bounded by the graph of the function *p*, the *x*-axis, and the lines x = 0.5 and x = 10.5.

Which approximation is better? Explain.