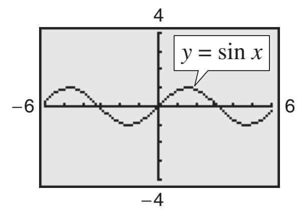
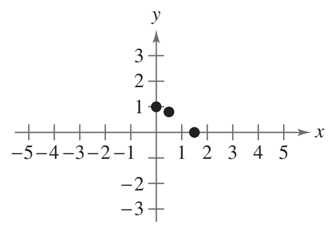
Chapter 11 Project *Tangent Lines to Sine Curves*

In this project you will develop a formula for the slope of the tangent line to the sine curve at an arbitrary point (x, y). In other words, you will derive a formula for the derivative of the function $f(x) = \sin x$. The first approach will be graphical, the second will be numerical, and the third will be algebraic.





a. ***Graphical Approach*** Use a graphing utility to graph the function $f(x) = \sin x$ as shown at the right. On a separate sheet of paper, plot your estimates of the *slope* of this curve at various x-values. For instance, some values of the slope underneath the sine curve have been plotted at the right. The slope is approximately 1 at $x = 0$, 0.8 at $x = 0.5$, and 0 at $x = 1.5$. After you have plotted 15 or 20 points, connect them with a continuous curve. Do you recognize this curve?

b. ***Numerical Approach*** The slope of the tangent line is given by the formula

$$m = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$

The difference quotient $[f(x + h) - f(x)]/h$ is a good approximation of the slope when h is small. If you choose $h = 0.01$, you have the following approximation for the slope of the sine curve at x.

$$m \approx \frac{\sin(x + 0.01) - \sin x}{0.01}$$

Use the *table* feature of a graphing utility to complete the table below. Plot these points and compare your results with part (a).

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
m									

c. ***Algebraic Approach*** To calculate the slope of the tangent line to $f(x) = \sin x$ algebraically, you need two trigonometric limits. (See Questions 1 and 2 below.)

Limit 1: $\displaystyle\lim_{h \to 0} \frac{\sin h}{h} = 1$ Limit 2: $\displaystyle\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

Use these limits to find a formula for the slope of the sine curve.

Questions for Further Exploration

1. Use a graphing utility to estimate Limit 1 in part (c) above *graphically* and *numerically*.

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

2. Use a graphing utility to estimate Limit 2 in part (c) above *graphically* and *numerically*.

$$\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

3. Follow the steps used in parts (a), (b), and (c) above to develop a formula for the derivative of the function $g(x) = \cos x$. Is it true that the sine and cosine functions are each other's derivatives? If not, which is the derivative of the other?

4. ***Amusement Park Ride*** An amusement park ride is constructed such that its height h in feet above ground in terms of the horizontal distance x in feet from the starting point can be modeled by

$$h(x) = 50 + 45 \sin \frac{\pi x}{150}, \quad 0 \le x \le 300.$$

(a) The formula for the derivative of $f(x) = a + b \sin cx$ is $bc \cos cx$. Use this to find the derivative of $h(x)$.

(b) Find the value of the derivative of $h(x)$ when $x = 50$, 150, 200, and 250.