Collaborative Project — Analytic Trigonometry

1. Show that each function at the left is equivalent to the corresponding function at the right. Then use a graphing utility to graph the function(s) at the right with the given viewing window and domain restrictions to sketch the figure described.

a. River branching into 2 streams (Viewing window: $0 \le x \le 3\pi$, $-50 \le y \le 50$)

$$f(x) = (\sin x + \cos x)(\tan x + \cot x) \rightarrow g(x) = \frac{1}{\cos x} + \frac{1}{\sin x}, \frac{\pi}{2} \le x \le 2\pi$$

b. Human finger with fingernail (Viewing window: $-3\pi \le x \le 3\pi$, $0 \le y \le 100$)

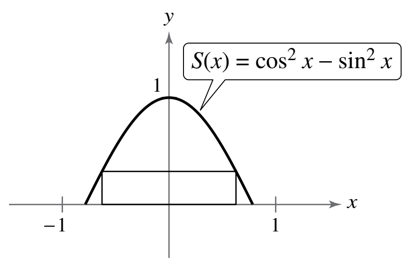
$$f(x) = 24 \sec^2 x - 20 \tan^2 x \rightarrow g(x) = \frac{4}{\cos^2 x} + 20, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$r(x) = 50 - 4 \csc\left(\frac{\pi}{2} - 2x\right) \rightarrow s(x) = 50 - \frac{4}{2\cos^2 x - 1}, -0.7 \le x \le 0.7$$

2. The vertical position (in inches) relative to the point of equilibrium of a weight on a spring is given by $f(x) = \cos(2x) - \sin(2x)$, where x is the time in seconds. Solve $f(x) = 1$ for $0 \le x < 2\pi$.

3. An overhead door is to be built on the front wall of a Quonset hut. The shape of the front wall is modeled by the graph of

$$S(x) = \cos^2 x - \sin^2 x, \; -\frac{\pi}{4} \le x \le \frac{\pi}{4} \text{ (see figure).}$$



The area of a rectangle inscribed in the graph of S is given by

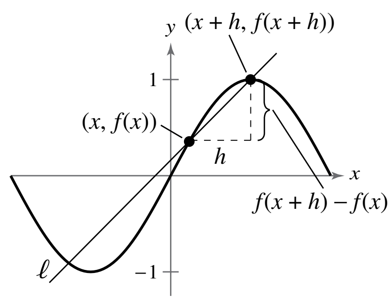
$$A(x) = 2x(\cos^2 x - \sin^2 x), \; 0 < x < \frac{\pi}{4}.$$

a. Rewrite the area formula using only one trigonometric function.

b. Approximate the maximum possible area of the inscribed rectangle.

c. Determine the values of x for which the area of the rectangle is greater than 0.5 square unit.

4. A line l can be drawn through two points $(x, f(x))$ and $(x + h, f(x + h))$ on the graph of $f(x) = \sin x$ (see figure).



a. The slope of line l is given by $m = \dfrac{f(x + h) - f(x)}{h}$.

Use $f(x) = \sin x$ to write another expression for m.

b. Show that your expression in part (a) is equal to

$$\frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}.$$

c. Use a graphing utility to graph $y = \dfrac{\sin x(\cos h - 1)}{h}$, $y = \dfrac{\cos x \sin h}{h}$, and $y = \cos x$ in the same viewing window for $h = 1$, $h = 0.5$, $h = 0.1$, and $h = 0.0001$.

d. Make a conjecture about the slope of the line through two points $(x, \sin x)$ and $(x + h, \sin(x + h))$ as h gets closer and closer to 0.