Collaborative Project – Exponential and Logarithmic Functions

1. **Experiment: Newton's Law of Cooling** Heat one cup of water. Place a thermometer in the water, and place the water (and thermometer) in a room or environment with a constant, unchanging temperature.

   a. Record the environment temperature E below. Use the table to record the temperature of the water every 5 minutes, starting at time $t = 0$.

   $E =$ ____

Time, t (minutes)	0	5	10	15	20	25	30
Temperature, T_t							

   b. Based on Newton's Law of Cooling, the temperature T of the liquid after t minutes is given by

   $$T(t) = E + (T_0 - E)e^{-kt}$$

   where k is a constant. Substitute the values for E, T_0, and T_{20} from part (a) into the equation and solve for k. (Substitute T_{20} for $T(t)$.) Then repeat the process to find k when $t = 5$, 10, and 15. Are the values of k reasonably close?

   c. Use the values of E, T_0, and k from parts (a) and (b) to rewrite the expression for $T(t)$. Use the value of k you feel will give the most accurate results. Explain your choice.

   d. Use the expression from part (c) to find $T(30)$. How does your answer compare to the actual temperature T_{30}?

   e. Use the expression from part (c) to predict the temperatures of the liquid when $t = 35$ and $t = 45$ minutes.

2. A patient is told to avoid caffeine for 8 to 12 hours before a blood test scheduled for 6 A.M. The blood test is reliable for up to 50 milligrams of caffeine in the bloodstream. The patient's body metabolizes caffeine at a rate of 13% per hour.

   a. At 10 P.M., the patient drinks a cup of coffee containing 150 milligrams of caffeine. Will the patient be ready for the blood test by 6 A.M.? Explain.

   b. How many milligrams of caffeine could the patient have ingested at 7 P.M. and been ready for the blood test at 6 A.M.?

3. A student comes to school with a highly contagious flu virus at a high school with 1030 students. The spread of the virus is modeled by

   $$P(t) = \frac{1030}{1 + 1029e^{-kt}}$$

   where P is the total number of students infected after t days.

   a. After 3 days, 121 students are infected. Complete the model $P(t)$ by solving for k.

   b. School policy is to close school when 40% of the students are infected. After how many days does this occur?

4. An athlete currently completes a 5-kilometer race in 26 minutes. The goal of the athlete is to complete the 5-kilometer race in 18 minutes. The model

   $$r(t) = 26 - 4.9 \log_{10}(t + 1), \quad 0 \le t \le 52$$

   describes the 5-kilometer race times $r(t)$, in minutes, of the athlete after t weeks of training.

   a. Can the athlete complete the 5-kilometer race in 20 minutes after 9 weeks of training? Explain.

   b. After how many weeks of training can the athlete achieve her goal? Round your answer to the nearest whole number.