Collaborative Project – Polynomial and Rational Functions

1. Creative Candles is testing the burn time of a hemisphere-shaped soy candle with a base diameter of 6 inches. Burning continuously, the candle does not tunnel, so its melted top surface is flat. The table shows the burning candle's height (in inches) every 2 hours.

Time, t	0	2	4	6	8	10	12	14	16
Height, h	3	2.24	1.90	1.39	1.18	0.97	0.78	0.59	0.41

a. Use a graphing utility to create a scatter plot of the data. Then use the regression feature of the graphing utility to find a quadratic model and a cubic model for the data. Graph both models with the data. Discuss how well each model fits the data.

b. Find the minimum value of the quadratic model. Does this function have any real zeros? Any complex zeros? Explain your reasoning.

c. Will the quadratic model accurately predict the candle's height after 20 hours? Explain.

d. Describe the left-hand and right-hand behavior of the graph of the cubic function.

e. Explain why the cubic function must have at least one real zero.

2. Creative Candles wants to design an open-top box with a volume V of at least 150 cubic inches that can hold any of several different candles. The box will be formed by cutting squares of length x from the corners of a rectangular piece of cardboard 12 inches wide and 15 inches long, and turning up the sides.

a. Find the domain of the function V based on the x-values that result in a box.

b. Write and solve a polynomial inequality to find all possible side lengths x of the squares that satisfy the company's volume requirement for the box.

3. A model for the relationship between the annual advertising expenses x and the profits P for Creative Candles is $P(x) = -0.2x^3 + 1.4x^2 + 11.4x - 20$, where x and P are both in tens of thousands of dollars. The company wants to expand its operations this year, while earning a profit of at least \$340,000.

a. Use a graphing utility to graph the function P. Use the graph to approximate the two annual advertising amounts that give the minimum amount of desired profit.

b. Explain how you can use synthetic division to verify the two advertising amounts in part (a). Then use synthetic division to verify the greater advertising amount.

4. Creative Candles is designing a rectangular gold plaque to hang in their show room. A rectangular etched region of the plaque x inches wide and y inches high must have an area of 88 square inches. There will be 2-inch borders at the top and bottom and 1-inch borders at the sides of the etched region.

a. Write a function for the area A of the plaque. Find the domain of the area function. Then suggest a reasonable domain for the width x of the etched region. Explain.

b. Find the asymptotes of the graph of $A(x)$.

c. What should the dimensions of the plaque be so that it covers the least amount of space on the wall?