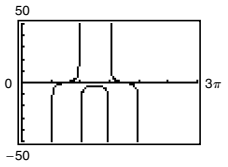
Analytic Trigonometry Answers

1. a. *Sample answer:*

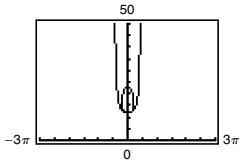
$$\begin{aligned}(\sin x + \cos x)(\tan x + \cot x) &= (\sin x + \cos x)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)\\ &= \frac{\sin^2 x}{\cos x} + \frac{\sin x \cos x}{\sin x} + \frac{\sin x \cos x}{\cos x} + \frac{\cos^2 x}{\sin x}\\ &= \frac{\sin^2 x}{\cos x} + \cos x + \sin x + \frac{\cos^2 x}{\sin x}\\ &= \frac{\sin^2 x + \cos^2 x}{\cos x} + \frac{\sin^2 x + \cos^2 x}{\sin x}\\ &= \frac{1}{\cos x} + \frac{1}{\sin x};\end{aligned}$$



b. *Sample answer:*

$$24 \sec^2 x - 20 \tan^2 x = 4 \sec^2 x + 20(\sec^2 x + \tan^2 x) = 4 \sec^2 x + 20 = \frac{4}{\cos^2 x} + 20;$$

$$50 - 4 \csc(\pi/2 - 2x) = 50 - \frac{4}{\sin(\pi/2 - 2x)} = 50 - \frac{4}{\cos(2x)} = 50 - \frac{4}{2\cos^2 x - 1};$$



2. $0, \pi, \frac{3\pi}{4}, \frac{7\pi}{4}$

3. a. $A(x) = 2x \cos(2x)$

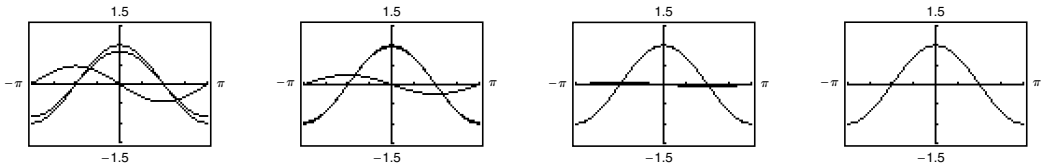
b. About 0.5611 square unit

c. Values of x from about 0.305 to about 0.549

4. a. $m = \dfrac{\sin(x + h) - \sin x}{h}$

b.
$$\begin{aligned}\frac{\sin(x + h) - \sin x}{h} &= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}\\ &= \frac{\sin x \cos h - \sin x + \cos x \sin h}{h}\\ &= \frac{\sin x(\cos h - 1)}{h} + \frac{\cos x \sin h}{h}\end{aligned}$$

c.



d. As h gets closer and closer to 0, the slope of the line through the points $(x, \sin x)$ and $(x + h, \sin(x + h))$ gets closer and closer to the value of $\cos x$.