

Collaborative Project – Exponential Functions

1. Experiment: Newton’s Law of Cooling Heat one cup of water. Place a thermometer in the water, and place the water (and thermometer) in a room or environment with a constant, unchanging temperature.

- a. Record the environment temperature E below. Use the table to record the temperature of the water every 5 minutes, starting at time $t = 0$.

$E =$ _____

Time, t (minutes)	0	5	10	15	20	25	30
Temperature, T_t							

- b. Based on Newton’s Law of Cooling, the temperature T of the liquid after t minutes is given by

$$T(t) = E + (T_0 - E)e^{-kt}$$

where k is a constant. Substitute the values for E , T_0 , and T_{20} from part (a) into the equation and solve for k . (Substitute T_{20} for $T(t)$.) Then repeat the process to find k when $t = 5$, 10, and 15. Are the values of k reasonably close?

- c. Use the values of E , T_0 , and k from parts (a) and (b) to rewrite the expression for $T(t)$. Use the value of k you feel will give the most accurate results. Explain your choice.
- d. Use the expression from part (c) to find $T(30)$. How does your answer compare to the actual temperature T_{30} ?
- e. Use the expression from part (c) to predict the temperatures of the liquid when $t = 35$ and $t = 45$ minutes.
- 2.** A patient is told to avoid caffeine for 8 to 12 hours before a blood test scheduled for 6 A.M. The blood test is reliable for up to 50 milligrams of caffeine in the bloodstream. The patient’s body metabolizes caffeine at a rate of 13% per hour.
- a. At 10 P.M., the patient drinks a cup of coffee containing 150 milligrams of caffeine. Will the patient be ready for the blood test by 6 A.M.? Explain.
- b. How many milligrams of caffeine could the patient have ingested at 7 P.M. and been ready for the blood test at 6 A.M.?
- 3.** A student comes to school with a highly contagious flu virus at a high school with 1030 students. The spread of the virus is modeled by

$$P(t) = \frac{1030}{1 + 1029e^{-kt}}$$

where P is the total number of students infected after t days.

- a. After 3 days, 121 students are infected. Complete the model $P(t)$ by solving for k .
- b. School policy is to close school when 40% of the students are infected. After how many days does this occur?