Collaborative Project — Matrices and Determinants

1. A toy company borrows a total of $155,000 in three loans at simple annual interest to develop three new toys. The amount borrowed for toy A is twice the amount borrowed for toy B. The sum of the amounts borrowed for toy A and toy B is $115,000 greater than the amount borrowed for toy C. The interest rate on the loan for toy B is 0.5% greater than that for toy A. The interest rate on the loan for toy C is 1.5% greater than that for toy A. The total annual interest for the three loans is $10,212.50

a. Set up a system of linear equations to determine the amounts borrowed for toy A, toy B, and toy C. Use matrices to solve the system by Gaussian elimination with back-substitution

b. Set up a system of linear equations to determine the interest rates for the amounts borrowed for toy A, toy B, and toy C. Use matrices to solve the system by Gauss-Jordan elimination.

2. The toy company manufactures 5 different action figures that are sold in two stores. Matrix P below shows the wholesale and retail price of each action figure and matrix I shows the inventory of each action figure that each store keeps on hand.

$$P = \begin{matrix} \\ \text{Wholesale} \\ \\ \\ \\ \\ \end{matrix} \overbrace{\begin{matrix} \text{Price} \\ \text{Retail} \end{matrix}} \begin{bmatrix} \$2.50 & \$10.00 \\ \$3.00 & \$11.00 \\ \$4.00 & \$14.00 \\ \$6.00 & \$17.00 \\ \$8.50 & \$20.00 \end{bmatrix} \begin{matrix} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{matrix} \} \text{Action figure}$$

$$I = \overbrace{\begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{matrix}}^{\text{Action figure}} \begin{bmatrix} 14 & 10 & 0 & 3 & 2 \\ 13 & 16 & 5 & 1 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix} \} \text{Store}$$

a. Find IP and interpret the result.

b. Each store doubles its inventory. Perform a matrix operation on I to show the result of this.

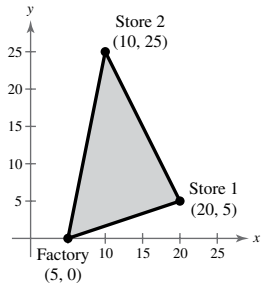
3. The table shows the numbers of dolls X, Y, and Z sold each month and the total sales for the months of September, October, and November. Use Cramer's Rule to solve the resulting system of equations. Interpret your results.

	X	Y	Z	Total Sales
September	30	70	20	$5550
October	60	78	91	$13,010
November	52	110	45	$10,050

4. The vertices of the triangle shown below represent the locations of the toy company's factory and its two outlet stores.

a. Find the area of the triangular region.

b. Find the equation of the line passing through the points representing Store 1 and Store 2.



5. The company's management team encodes the names of employees when using email to discuss sensitive information. Each member of the management team has a computer program that uses the matrix A to encode row matrices or the matrix A^{-1} to decode row matrices, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}.$$

a. Find the matrix A^{-1}. Then use matrix A^{-1} to decode the names in the following message.

PROMOTIONS RECOMMENDED FOR

[127 163 40] [88 125 74] [3 1 29] [64 105 109] [20 48 92]

[58 81 43] [44 77 93] [25 42 48] [92 137 102] [89 137 117]

b. Use matrix A to encode the names in the following message.

PLEASE REVIEW PENSION PACKAGE FOR

Stephany Kelley

Ramone Whitehead