Collaborative Project — Topics in Analytic Geometry

A high school has submitted a request for upgrades to its football complex in time for their annual rivalry game.

1. The field is designed with a parabolic surface so that rain may drain off more effectively. One particular design calls for the field to be 160 feet wide and 0.5 foot high at its center.

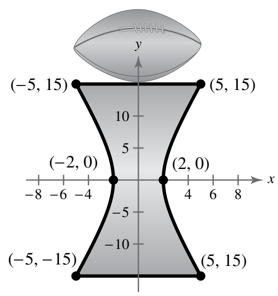
a. Find an equation of the parabola with its vertex on the y-axis that models the field surface such that y is the height of the surface.

b. How far from the center of the field is the surface 0.3 foot lower than in the middle?

2. The design for the entrance from the team locker room onto the field is a parabolic archway, 10 feet high at its center. At a height of 7 feet the archway is 12 feet wide. How wide is the archway at ground level?

3. The hallway will have a semielliptical mural, 10 feet high at the center and 22 feet wide along the base. The contractor plans to sketch the boundary of the mural on the wall using two nails (located at what will be the foci of the boundary) and a string with an end tied to each nail. Determine the positions of the nails and the length of string.

4. The base of the trophy for the rivalry game has a hyperbolic cross section (see figure).



a. Write an equation that models the curved sides of the trophy.

b. Each unit in the coordinate plane represents 1 inch. Find the width of the trophy at a height of 12 inches from the base.

Every year, the school brings in a Kid Zone for kids to play in while their parents watch the rivalry game. The Kid Zone consists of a Ferris wheel, a Scrambler, and a to-scale football field where kids can play at their leisure.

5. When the Ferris wheel is projected onto the polar coordinate system, its center is located at the pole and all distances are measured in feet. Passengers enter a car at $(20, 3\pi/2)$. The wheel takes 40 seconds to complete one counterclockwise rotation.

a. Write a polar equation that models the possible positions of a passenger car.

b. Find and interpret the coordinates of a passenger car after 25 seconds of rotation.

c. Convert the point in part (b) to rectangular coordinates. Interpret the coordinates.

6. The Scrambler whips passengers around in the approximate shape of the rose curve $r = 25 \sin 5\theta$.

a. Find the maximum distance a passenger car will be from the center.

b. Find the angles at which the cars are at the maximum distance from the center.

c. Sketch the graph of the polar equation.

7. The football field in the Kid Zone is scaled such that 1 "yard" equals 1.5 feet. A child stands at the "10-yard" line of the field, ready to punt a football to his friend. He punts the ball with an initial velocity of 65 feet per second and at an angle of elevation of 23°.

a. Write a set of parametric equations that represent the velocity of the football.

b. How long (in seconds) is the ball in the air?

c. Assuming the football travels along a path perpendicular to the "10-yard" line, where does the ball land?