Extension

Magnitude of a Scalar Multiple

The magnitude (or length) of a vector $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}.$$

By definition, the magnitude of a vector cannot be negative. The same is true for the magnitude of a scalar multiple of a vector.

Magnitude of a Scalar Multiple

Let $\mathbf{v}$ be a vector and let c be a scalar. Then

$$\|c\mathbf{v}\| = |c|\|\mathbf{v}\|.$$ $|c|$ is the absolute value of c.

EXAMPLE 1 Finding the Magnitude of a Scalar Multiple

Let $\mathbf{u} = \langle 1, 3 \rangle$ and $\mathbf{v} = \langle -2, 5 \rangle$. Find the magnitude of each scalar multiple.

a. $\|2\mathbf{u}\|$

b. $\|-5\mathbf{u}\|$

c. $\|3\mathbf{v}\|$

Solution

a. $\|2\mathbf{u}\| = |2|\|\mathbf{u}\| = |2|\|\langle 1, 3 \rangle\| = |2|\sqrt{1^2 + 3^2} = 2\sqrt{10}$

b. $\|-5\mathbf{u}\| = |-5|\|\mathbf{u}\| = |-5|\|\langle 1, 3 \rangle\| = |-5|\sqrt{1^2 + 3^2} = 5\sqrt{10}$

c. $\|3\mathbf{v}\| = |3|\|\mathbf{v}\| = |3|\|\langle -2, 5 \rangle\| = |3|\sqrt{(-2)^2 + 5^2} = 3\sqrt{29}$

Exercises

Finding the Magnitude of a Scalar Multiple **In Exercises 1–6, find the magnitude of the scalar multiple, where $\mathbf{u} = \langle 1, 0 \rangle$, $\mathbf{v} = \langle 5, -3 \rangle$, and $\mathbf{w} = \langle -4, -2 \rangle$.**

1. $\|6\mathbf{u}\|$ **2.** $\left\|\frac{1}{2}\mathbf{u}\right\|$ **3.** $\|-7\mathbf{v}\|$

4. $\|\pi\mathbf{v}\|$ **5.** $\left\|\frac{3}{4}\mathbf{w}\right\|$ **6.** $\|-8\mathbf{w}\|$