Chapter 5 Project *Projectile Motion*

Parametric equations are equations that represent two variables in terms of a third variable, called the parameter. In this project, you will use parametric equations to model the path of a projectile. These parametric equations represent position x and position y in terms of time t. For any time t, the horizontal position $x(t)$ and vertical position $y(t)$ of a projectile (ignoring air resistance) launched at ground level is given by the parametric equations

$$x(t) = (v_0 \cos \theta)t$$

$$y(t) = (v_0 \sin \theta)t - 16t^2.$$

In these equations, θ is the angle with the horizontal and v_0 is the initial velocity in feet per second, as indicated in Figure 1.

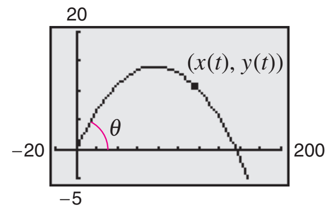

Figure 1

Set your graphing utility to *parametric* and *degree* modes, and use the viewing window shown in Figure 2. Let $v_0 = 88$ feet per second and $\theta = 20°$, and graph the parametric equations

$$x(t) = (88 \cos 20°)t$$

$$y(t) = (88 \sin 20°)t - 16t^2.$$

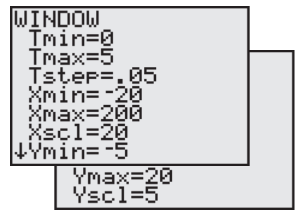

Figure 2

Use the *zoom* and *trace* features of your graphing utility to (a) find the maximum height attained by the projectile, (b) find the time at which the maximum height occurs, (c) determine the length of time that the projectile is in the air, and (d) determine the range of the projectile.

Questions for Further Exploration

1. Verify algebraically the range of the projectile by solving the equation $(88 \sin 20°)t - 16t^2 = 0$ for t and then evaluating $x(t)$ at this t-value.
2. Use a graphing utility to find the maximum height and range of the projectile when $\theta = 30°$ and $v_0 = 132$ feet per second.
3. Let $v_0 = 60$ feet per second. Find the range for the angles $\theta = 20°, 30°, 40°, 50°$, and $60°$. In general, what angle should you use to produce the maximum range?
4. What is the relationship between the time the projectile reaches its maximum height and the time it takes for the projectile to return to the ground? Explain.
5. Eliminate t from the parametric equations

   $$x(t) = (v_0 \cos \theta)t \quad \text{and} \quad y(t) = (v_0 \sin \theta)t - 16t^2$$

   by solving for t in the first equation and substituting this value into the equation for y. Show in this case that the height of the projectile is given by the equation

   $$y = (\tan \theta)x - \frac{16 \sec^2 \theta}{v_0^2}x^2.$$

   Use this equation to find the angle θ corresponding to a range of 200 feet and initial velocity of $v_0 = 80$ feet per second.