Chapter 8 Project *Fitting Models to Data*

In this project, you will explore difference quotients and see how they can be used to find average rates.

You are driving from Atlanta to Miami. The trip is about 660 miles and takes you 12 hours. Your average speed is

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{660}{12} = 55 \text{ miles per hour.}$$

The concept of average rate of change can be generalized as follows. Let f be a function defined on the interval $[a, b]$. The **average rate of change of f** from a to b is

$$\text{Average rate of change} = \frac{f(b) - f(a)}{b - a}.$$

This expression is called a *difference quotient*.

a. Calculate the average rate of change of $f(x) = 3x + 4$ on the interval $[2, 6]$. Select any other interval and show that you obtain the same average rate of change.

b. Calculate the average rates of change of $f(x) = x^2$ on the intervals $[1, 3]$ and $[4, 6]$. Are they equal? Explain.

c. During a one-hour trip, your average speed is 50 miles per hour. Discuss the relationship between this speed and the speeds shown on your speedometer.

Questions for Further Exploration

1. Let $f(x) = x^2 - 2$. Calculate the average rates of change of f on the intervals $[1, 3]$, $[1, 2]$, $[1, 15]$, and $[1, 1.1]$.

(a) Find an expression for the average rate of change of f on the interval $[1, 1 + h]$, where h is any real number.

(b) Create a table that shows the average rate of change of f for several values of h. Choose values of h that get closer and closer to 0. What value does the average rate of change of f approach as h approaches 0?

(c) The answer to part (b) is the slope of the tangent line to the graph of f at the point $(1, f(1))$. Use a graphing utility to graph this line and f in the same viewing window. Describe the behavior of the graphs near the point $(1, f(1))$.

2. The data in the table shows the costs of first-class postage (for the first ounce) in the United States for selected years from 1978 to 2006, where t represents the year, with $t = 8$ corresponding to 1978. (Source: U.S. Postal Service)

t	8	11	15	18	21
Postage	\$0.15	\$0.20	\$0.22	\$0.25	\$0.29

t	25	29	31	32	36
Postage	\$0.32	\$0.33	\$0.34	\$0.37	\$0.39

(a) Find the average rate of change of the cost of postage between each two adjacent time intervals.

(b) When was the average rate of change largest? When was it smallest?

(c) Are there intervals over which the average rate of change was zero? What does this mean?