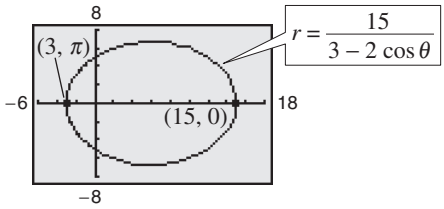
Chapter 9 Project *Polar, Rectangular, and Parametric Forms*

In this project, you will compare the polar, rectangular, and parametric forms of equations for conics.

a. Consider the polar equation

$$r = \frac{15}{3 - 2\cos\theta}.$$



In this chapter, you learned that the graph of this polar equation is an ellipse and that one of the ellipse's foci is at the pole, as shown in the graph above. To find the rectangular equation for this ellipse, begin by rewriting the equation as

$$3r - 2r\cos\theta = 15.$$

Then use the substitutions $r = \sqrt{x^2 + y^2}$ and $r\cos\theta = x$ to find the rectangular equation. After you find the rectangular equation, write it in the standard form

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1.$$

b. Use the standard form of the ellipse in part (a) to find the center, foci, and eccentricity of the ellipse. Use the *function* mode of a graphing utility to graph the ellipse. Compare your graph with the graph above.

c. Use the standard form of the ellipse and the identity $\sin^2\theta + \cos^2\theta = 1$ to write parametric equations for the ellipse. Then use the *parametric* mode of a graphing utility to graph the ellipse.

Questions for Further Exploration

1. Sketch the graph of the polar equation

$$r = \frac{32}{3 + 5\sin\theta}.$$

Which type of conic is this?

2. Write the standard form of the rectangular equation of the conic in Question 1. Then graph the equation using the *function* mode of a graphing utility. Does your graph agree with the graph obtained in Question 1? Which of the two graphs is easier to obtain? Explain your reasoning.

3. Write parametric equations for the conic in Question 1. Then use the *parametric* mode of a graphing utility to graph the conic. Does your graph agree with the graph obtained in Question 1?

4. Consider the parametric equations

$$x = 4 + 5\cos t \quad \text{and} \quad y = 3\sin t.$$

With appropriate scaling of the coordinate system, could these equations represent the motion of a comet about the sun? Explain your reasoning.