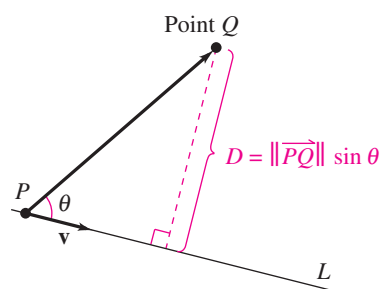# Chapter 10 Project *Distance Between a Point and a Line*

Previously, you learned how to find the distance between a point and a plane. A related problem is to find the distance between a point and a *line*. You will see that the formula resembles that for the distance between a point and a plane, except that you replace the dot product with the cross product and the normal vector **n** with a direction vector **v** for the line.

In order to develop this new formula, suppose you are given a line L and a point Q not on the line. Let D be the distance between the point and the line, as shown in the figure at the right. Let $\mathbf{v} = \langle a, b, c \rangle$ be the direction vector for L and select any point P on the line. Then $D = \|\overrightarrow{PQ}\| \sin \theta$ where θ is the angle between **v** and $\overrightarrow{PQ}$. Using the second geometric property of the cross product ($\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\| \sin \theta$), you know that

$$\|\mathbf{v}\|\|\overrightarrow{PQ}\| \sin \theta = \|\mathbf{v} \times \overrightarrow{PQ}\| = \|\overrightarrow{PQ} \times \mathbf{v}\|$$

Consequently,

$$D = \|\overrightarrow{PQ}\| \sin \theta = \frac{\|\mathbf{v}\|\|\overrightarrow{PQ}\| \sin \theta}{\|\mathbf{v}\|} = \frac{\|\overrightarrow{PQ} \times \mathbf{v}\|}{\|\mathbf{v}\|}.$$

Questions for Further Exploration

1. Show that the distance between the point $Q = (3, -1, 4)$ and the line given by $x = -2 + 3t$, $y = -2t$, and $z = 1 + 4t$ is $\sqrt{6}$.

2. Find the distance between the point $(10, 3, -2)$ and the line through the points $(-2, 3, 1)$ and $(2, 3, 0)$.

3. Find the distance between the two parallel lines $x = 3t - 1$, $y = -t + 2$, $z = 4t$, and $x = 6t + 2$, $y = -2t$, $z = 8t - 3$.

4. Find the distance between the following two skew lines. (*Skew lines* are lines that are neither parallel nor intersecting.)

$$x = \frac{y}{2} = \frac{z}{3}$$

$$\frac{x-1}{-1} = y - 4 = z + 1$$

5. The *slant height* of a regular pyramid is the distance along a face of the pyramid from the base to the vertex, as shown in the figure.
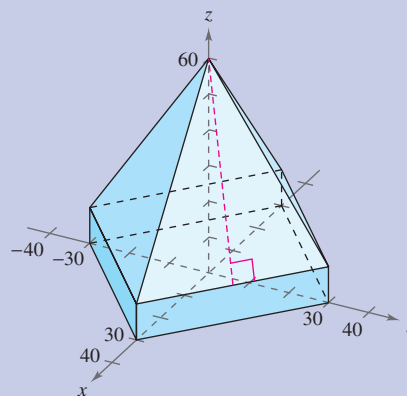
Find the slant height of a pyramid whose base lies along the line given by $x = -30t$, $y = 30 + 30t$, $z = 10$ and whose vertex is at the point $(0, 0, 60)$.